

Math 7110 – Homework 1 – Due: September 1, 2021

Practice Problems:

Problem 1. Show that an element in S_n has order two if and only if it is a product of commuting cycles of length 2.

Problem 2. Show that $GL_n(F)$ is non-abelian if $n \geq 2$.

Problem 3. Show that the following group is isomorphic to the dihedral group D_4

$$\langle x, y \mid x^2 = y^2 = (xy)^2 = 1 \rangle.$$

Problem 4. Make an appointment to visit me in my office sometime before 5 PM on Thursday, September 2. I will pass around a signup sheet for meetings on Wednesday, August 25th.

Be prepared to answer the following questions:

- (1) Why are you taking this class and what do you hope to get out of it?
- (2) What other math classes have you taken?
- (3) What are your career goals after you graduate?

Type solutions to the following problems in L^AT_EX, and email the tex and PDF files to me at dbernstein1@tulane.edu by 10am on the indicated date. Please title them as [lastname].tex and [lastname].pdf. When preparing your solutions, you must follow the rules as laid out in the course syllabus.

Graded Problems:

Problem 5. Answer the following questions:

- (1) If $\phi : G \rightarrow H$ is an isomorphism, prove that $|\phi(x)| = |x|$ for all $x \in G$.
- (2) Is the result true if ϕ is just a homomorphism? Prove or give a counterexample.
- (3) Given $x \in S_n$, show that $|x|$ is the least common multiple of the lengths of the cycles in its cycle decomposition.
- (4) Are S_4 and D_{24} isomorphic? Why or why not?

Problem 6. Let G be a group and let $\text{Aut}(G)$ be the set of all isomorphisms from G onto G .

- (1) Prove that $\text{Aut}(G)$ is a group under function composition.
- (2) Let G be a finite group and let $\sigma \in \text{Aut}(G)$ be such that $\sigma(g) = g$ if and only if $g = 1$. If σ^2 is the identity map, prove that G is abelian.